Taken directly from Bejan Chapter 2 problems

The Japanese teapot *kyusu* has a hollow handle that feels cool to the touch even when the pot contains boiling water. This handle can be modelled as a hollow-cylinder fin, as shown on the right side of the figure, where

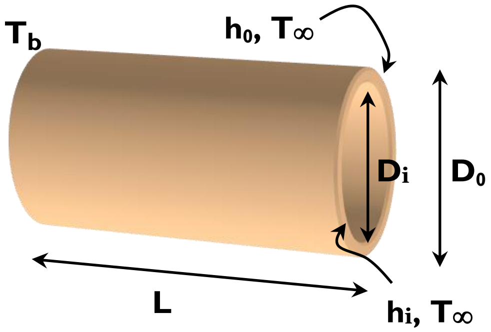
D_0 = 2 cm	h_0 = 10 W/m^{2}K	T_b = 50°C
D_i = 1.5 cm	h_i = 2 W/m^{2}K	T_∞ = 15°C
L = 7 cm		

The handle is made out of porcelain. The heat transfer coefficient on the outer surface of the handle is considerably larger than on the inner surface, because the buoyancy-driven air flow is more intense around the handle.

a) calculate the appropriate parameter *m* of the hollow-cylinder fin and show that this fin is a "long fin". (k=1.03 W/mK)

b) calculate the distance from the base of the handle where the handle temperature is 30°C

c) calculate the instantaneous heat transfer rate through the base of the handle.



a) In this case the surface area and cross sectional area are defined differently.

$$m = \sqrt{\frac{hp}{kA}} = \sqrt{\frac{h_iP_i + h_oP_o}{kA}} = \sqrt{\frac{2\pi D_i + 10\pi D_o}{k\pi\left(\frac{D_0^2}{4} - \frac{D_i^2}{4}\right)}} = 71.4\, m^{-1}$$

For a long fin the temperature along the fin is given by

$$\theta = \theta_0 e^{-mx}$$

Putting in the values we get:

$$\theta = (50 - 15)\, e^{-71.4*0.07} = 0.236^oC$$

Since temperature from the end of the fin is virtually the same as the air, the fin can be considered long.

b) Look for a particular temperature difference between the air at 15C and the handle which has dropped to 30 C and identify distance x.

$$\theta = (30 - 15) = 15e^{-71.4*x} \rightarrow x = -\frac{\log\left(\frac{15}{35}\right)}{71.4} = 0.012\, m$$

This shows that the handle has cooled to 30 C after 1.2 cm.

c) The temperature difference for a long fin is given by:$\theta = \theta_0 e^{-mx}$

The temperature gradient is therefore the differentiation of this.

So temperature gradient at location x is given by: $\frac{d\theta}{dx} = -m\, \theta_0 e^{-mx}$

From Fourier's law, the heat flow due to a temperature gradient is given by

$$\dot{q} = -kA\frac{d\theta}{dx}$$

The heat transfer rate through the base of the handle (x=0) is therefore

$$\dot{q} = 1.03\frac{\pi}{4} * (0.02^2 - 0.015^2)\,(71.4 * 35) = 0.35\, W$$

So heat flow through handle (fin) is 0.35 W, or small.

Extra: We can use the heat flow to calculate the efficiency and the effectiveness of the fin.

Effectiveness is the heat flow from the fin/ heat flow that would have happened if there was no fin.

If there was no fin then the heat loss from the area of the base of the fin would have been

$$\dot{q}_{no-fin} = h_o A_{base}(T_b - T_\infty) = 10\,\pi\left(\frac{(0.02)^2}{4} - \frac{(0.015)^2}{4}\right)(50 - 15) = 48mW$$

The heat flow from the surface of the fin is equal to the heat flow through the base (or where else will it go).

So the effectiveness. ϵ is

$$\epsilon = \frac{0.35}{0.048} = 7.29$$

So effectiveness ϵ = 7.3, and heat flow from fin can also be written as:

$$\dot{q} = \epsilon h_o A_{base}(T_b - T_\infty)$$

Similarly we can calculate efficiency of fin design in a similar manner. The efficiency η of a fin is given by the ratio of the actual heat loss from the fin/ the heat loss if the fin was all at the base temperature.

The heat loss from the fin if it was at the theoretical maximum $\dot{q}_{Max}$ is given by:

$$\dot{q}_{Max} = h_o A_{surf}(T_b - T_\infty)$$

The Area of the fin is the area of the end plus the area inside and outside of the fin and can be given by:

$$A_{surf} = \pi\left(\frac{D_0^2}{4} - \frac{D_i^2}{4}\right) + L\pi D_0 + L\pi D_i = \frac{\pi}{4}(0.02^2 - 0.015^2) + \pi(0.07)(0.02 + 0.015)$$
$$= 7.83 \times 10^{-3}\ m^2$$

So the theoretical maximum heat flux from the fin $\dot{q}_{Max}$ is given as

$$\dot{q}_{Max} = 10(7.83 \times 10^{-3})(35) = 2.7405\ W$$

Again the actual heat loss from the fin is the heat loss through the base (it might be worth your while to remember the method for calculating this), so the efficiency of the fin is:

$$\eta = \frac{\dot{q}}{\dot{q}_{Max}} = \frac{0.35}{2.7405} = 0.1277$$

In other words, the efficiency of this fin is $\eta = 0.13$, or 13%. This is not surprising since the fin is long. The temperature difference for the fin tip is small and so little heat is lost at the tip. Does that mean that a short fin is better? That depends on the fin, since a lot of a little can be better that a little of a lot. In other words little heat is lost from the tip, but there is a lot of fin present and even the little loss adds up if the fin is longer.

If we now know the efficiency of the fin, then we can use this to calculate the heat loss from the fin.

$$\dot{q} = \eta(h_o A_o + h_i A_i)(T_b - T_\infty)$$

They all ties together and this is worth knowing for complex fin shapes where we can measure the heat flux into the system more easily that some other parameters.

Like everything else that we see from the first moments of conscious life, hair and fur are things that we take for granted. Their purpose seems obvious not only with respect to our own needs, but also for the survival of numerous other species. The insulation effect of hair can be explained through the argument that when the hair strands are sufficiently dense, they trap a blanket of air in the tight spaces created between the strands. Well known for its low thermal conductivity, this air blanket insulates the warm skin against the colder atmosphere.

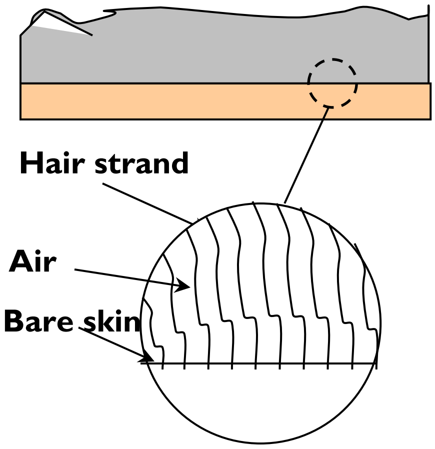
The hair-covered surface is considerably more interesting when approached from the point of view – the function – of the individual hair strand. The thermal conductivity of hair material (roughly that of human skin, 0.37 W/mK) is 14 times greater than that of ambient air. It seems that one effect of the hair strands is to "extend" the warm surface into the cold atmosphere, in the same way that a population of cylindrical spines (pin fins) extends the heat transfer surface of a heat exchanger. This finning effect is, most certainly, an unwanted by-product of the true function of the hair strands, which is to slow down the breeze that would otherwise blow by.

The fur of a lynx, for example, has a density of 9,000 strands/cm^2 and a cylindrical hair strand with a diameter of 27×10^{-6} m. The heat transfer coefficient between the hair strand and the surrounding air is 100 W/m^{2}K, and the heat transfer coefficient between the bare portions of the skin and the same air flow is 10 W/m^{2}K. The temperature difference between the skin surface and the air flow happens to be 10K. The fur-covered area under consideration is a square with side 10 cm.

a) calculate the heat transfer rate that leaves the skin through the roots of all the hair strands. Treat each hair strand as a long fin.

b) calculate the heat transfer rate released by the bare portions of the skin. Compare this contribution with the heat transfer current calculated in part a)

c) estimate the order of magnitude of the distance of conduction penetration along the hair strand. In other words, for what length near its root is the hair strand significantly warmer than the surrounding air?



a) The flow of heat from a long fin is given by:

$$\dot{q} = -kA\frac{d\theta}{dx} = m\theta_0 kAe^{-mx}$$

We need to calculate m;

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{100\frac{\pi(27E-6)}{0.37\frac{\pi(27E-6)^2}{4}}} = 6328\,m^{-1}$$

So heat loss is:

$$\dot{q} = \frac{0.37(6328)10\pi(27E-6)^2}{4}\,e^0 = 1.34E-5\,W$$

This is per hair. There are 9000 strands /cm^2 and if we consider a 100 cm^2 area, then there are 900000 strands.

So total heat loss through all hairs is:

$$\dot{q} = 1.34E-5 * 900000 = 12.07\,W$$

b)

Area of a single hair is $\frac{\pi(27E-6)^2}{4} = 5.73E-10\,m^2$

900000 strands have an area of 0.000515 m^2

The area we are looking at is $100\,cm^2\ or\ 0.01\,m^2$

The area of skin is therefore $0.01 - 5.15E-4\,m^2 = 0.0095\,m^2$

The heat loss from the bare portion of the skins is then

$$\dot{q} = hA\Delta T = 10(0.0095) * 10 = 0.9485\,W$$

This is less heat loss than from the hairs.

c) For a long fin the temperature difference is given by $\theta = \theta_0 e^{-mx}$

If we define the length of the hair that is significantly warmer than the air as when the flow is 0.1C warmer, then the exponential term must be equal to 0.1.

$$10 * e^{-mx} = 0.1 \rightarrow x = \frac{-\log(0.01)}{m} = 7.27E-4\,m\ or\ 0.7\,mm$$

In other words the temperature of the hair drops quickly along its length (hence it can be modelled by a long fin)

Let us put a Thermoelectric Generator (TEG) in the wall of a chimney. The exhaust gas is at 500C. Assume the heat transfer coefficient would be 20 W/m^{2}K between the exhaust gas and the surface of the TEG. The TEG has a size 5 cm x 5cm and the surface is at 250 C.

a) What will be the heat flow between the exhaust gas and the TEG?
b) We wish there to be a heat flow of 100W through the TEG if it is to generate 1 W of electrical power. We can increase the surface area to 0.1 m x 0.1 m if the base of the extended surface is thick enough. What effectiveness does this correspond to for an extended surface?

a) The area of the TEG is $0.05\,x0.05 = 2.5 \times 10^{-4}\,m^2$. The heat flow is therefore

$$\dot{q} = hA_{TEG}(T_{exhaust} - T_{TEG}) = (20)(2.5 \times 10^{-4})(500 - 250) = 1.25W$$

b) The heat flow through an extended surface is related to the effectiveness ϵ and the base area by the expression $\dot{q} = hA_{base}\epsilon(T_{exhaust} - T_{TEG})$. Rearranging and adding the numbers we get:

$$\epsilon = \frac{(100W)}{20Wm^{-2}K^{-1}\,(1 \times 10^{-2})(500 - 250)} = 2$$

In other words the extended surface would have to be 2 times more effective than the surface of the TEG to increase the heat flow from 1.25 W to 100W.

The heat flow can be define by reference to the effectiveness and the efficiency.

$$\dot{q} = hA_{base}\epsilon(T_{exhaust} - T_{base}) = h(\eta A_{fin} + A_{if})(T_{exhaust} - T_{base})$$

So

$$\eta = \frac{\epsilon A_{base} - A_{if}}{A_{fin}}$$